PREVIEW

A treatise on the differentials and integrals

Or: *why is IDD proposed for fatigue life assessment under general (any) kind of loading including multiaxial, non-proportional and non-cyclic loading*?

Mankind had been at a standstill in science and technology for millennia in succession by year 1600, i.e. by 17th century. *This related to non-development of mathematics*. People could not go far beyond addition, subtraction, multiplication and division, and beyond the notions of elementary geometry, planimetry and stereometry.

There was, though, guesswork of calculus (differential and integral calculation) while fragmenting a volume or an area into fragments less and less (also for finding the gravity center). Archimedes (287 – 212 BC) came close to the idea of differentiation and integration. However, in the next two millennia each problem concerning area, volume, gravity center, floating, levers, simple engineering structures, etc. was solved for itself in a specific, particular way, without any general mathematical approach.

Neither was the velocity realized as a derivative (with respect to time) of a changing object's position in motion mechanics; nor was it known that an integral of the velocity gives the position; nor that the acceleration is a derivative of the velocity and that the acceleration is namely proportional to the force. Variable velocity, variable acceleration and a variable in general remained without a clear mathematical definition since *the idea of defining a varying function within an infinitesimal interval (differential) of argument's variation* was missing. Thus, the mechanics was at a standstill and hence the physics was not developed. Prior to year 1600 the mankind did not suspect at all of future scientific and technological miracles: machines, electricity, telephone, telecommunications, radio, television, nuclear power, astronautics, telecommunication satellites, computers …

What actually happened to the human being's thinking after 1600 so that, in four centuries only comparatively to millennia of standstill, a lot of sciences suddenly progressed and enabled the above achievements and miracles?

What happened in 17th century is that *the idea of infinitely little quantities (infinitesimals) was carried out*. *This was an infinitely great jump of the mankind*. The human being's thinking readjusted to search of relations on differential level i.e. among infinitely small differentials of functions and their arguments. The mechanical notion of velocity was realized as a quotient of dividing displacement by time differential. In general, the quotient of dividing a function differential $\mathrm{d}F(x)$ by argument differential $\mathrm{d}x$, i.e. $f(x) = \mathrm{d}F(x)/\mathrm{d}x$, called derivative of the function $F(x)$, attained primary importance.

The mankind learned how to differentiate for finding the derivative and the relating differentials. The derivative is the most important characteristic of the change of a function: the rate of this change. It represents speed, velocity, intensity, density, slope, stress etc. *The human being learned also how to integrate i.e. to sum the differentials* $\mathrm{d}F(x) = f(x)\mathrm{d}x$ *of infinite number* in order to obtain the integral function (the primitive) $F(x)$.

Isaac Newton, 1642 – 1727, was the father of the calculus. The other founder is Gottfried Leibniz, 1646 –1716. He finally formed the terms and symbols for differential, integral, derivative, differential calculation, differential equation, coordinates, function, algorithm, etc. [55]. The basis of integration is given by the famous Newton-Leibniz theorem (equation) for integrating within fixed limits from a to b:

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a)\,. \tag{0-1}$$

he upper limit can also be variable (running, current). If it is denoted as $x_{\max}$, then

$$\int_a^{x_{\max}} f(x)\mathrm{d}x = F(x_{\max}) - F(a)\,. \tag{0-2}$$

It is remarkable in Eq. 0-2 that $F(x_{\max})$ is namely the primitive $F(x)$ function as $x_{\max}$ assumes the role of x. This is a key interpretation for the present thesis (Section 2.3.1).

After the fundamental Newton-Leibniz theorem, what was revolutionarily developed is *the general and universal mathematical way for obtaining relations searched among variables*: *namely as integral results from integration of relations found on a differential level*, *under any integration conditions*. Into the root of every science, differential relations (differential equations) are namely set since only in this way the functions as variables can be defined: they are considered as constants within the frames of arguments' differentials thanks to neglecting infinitely little differences of higher order. Thus, for example, a curved line is substituted by a broken line having straight infinitesimal segments; an area beneath a curve is substituted by a stepped polygon having infinitely short steps; the infinitely small planes of an infinitesimal rectangular parallelepiped (cubic volume) of a deformable body remain planar although finite planes of the body warp; and so on, and so on.

But the main advantage of setting relations among differentials in the basis of each science is the following: these relations are independent of the integration conditions, and hence integral results under arbitrary integration conditions can be obtained.

To clarify this, let the problem of the volume of a body having an arbitrary shape be called. Prior to 1600, trials were mostly done to fragment a body into rectangular parallelepipeds of finite number and of finite dimensions a, b, c. From the contemporary point of view, this can be interpreted so: *adapting* the solution $V = abc$ to different integration conditions of different shapes of bodies. And, a recommendation will immediately follow: to build an integral of differentials $\mathrm{d}V$ introduced (for example) as $\mathrm{d}V = z(x,y)\mathrm{d}x.\mathrm{d}y$ where $y = y(x)$ is the equation of contour of section of the body with the plane $z = 0$. The method is uniform: an integral method for obtaining a volume; the integration conditions i.e. the input functions $z = z(x,y)$ and $y = y(x)$ are only different for the different shapes of bodies.

Another (more specialized) example: the general approach of deformable-body mechanics is dividing into infinitesimal rectangular parallelepipeds of infinite number. On such a differential level there are 'caught' the relations (differential equations) among the differentials of stresses on any parallelepiped, also among its strains and displacements. Then, researchers integrate over all the parallelepipeds under *arbitrary shape of the body and arbitrary loads*.

It becomes apparent that *if the differential and integral approach is not applied to a scientific field, then there will not be a uniform, all-acknowledged and universal method in a general formulation of the problem i.e. under general integration conditions*. Instead, there will be: hundreds of methods proposed in particular formulations, i.e. individual results under conditions which would have been particular integration conditions if researchers had integrated; hundreds of attempts to carry particular solutions onto a higher level of generalization what actually are trials to inductively adapt results from simpler integration conditions to more complicated ones; thousands of written papers resulted from scattering efforts.

Not every problem of integration, respectively of solving differential equations, has its analytical solution. It depends on the complexity of the equation and the integration conditions. But the latter-day mass computerization and the new and new kinds of software enabled the numerical integration to dominate. Infinitesimal differentials or elements are substituted by finite ones. Respectively, in case for some scientific problems the analytical integration under arbitrary integration conditions was unthinkable time ago and therefore the universal mathematical approach from differentials to integrals was not applied, nowadays it is not a problem to do numerical integration by means of a computer.

Hence, if in a scientific field there were methods developed without differentials and integrals, then the accumulated experience should be readjusted onto differential level and integration from it. This could develop the field as revolutionarily as the calculus developed the mathematics and all related contemporary sciences.

Now it is to answer the initial above question why IDD is proposed for fatigue life assessment under general (any) kind of loading: because namely in this field of research the general deductive mathematical way from differentials (of fatigue damage) to an integral had not been applied, namely under general (any) integration conditions (of loading). Instead, researchers went inductively from particular solutions to adapting them in a more general formulation, in many different scattering ways, without reaching a general and uniform method. To happen so was for historical and technological reasons (the future computers were still missing). Yet it is the high time to try with fatigue damage differentials and their numerical integration.

Retrospection of IDD

For taking up an exact attitude towards the proposed IDD method, it is of importance to know how the method was initiated and developed, and what a reception it had. Besides, the Preview's part below is a part of the literature review and the introduction to IDD.

The author graduated 1972 from the Technical University of Sofia as a Master of Mechanical Engineering with veneration for the differential and integral approach. That is why the next logical step was applying and qualifying for the so-called "Block B" of the same Technical University of Sofia: one-year specialization in Applied Mathematics. That "Block B" was newly founded by people who believed that engineers having mathematical aptitude and additional mathematical training would better push the engineering sciences. Under such setting from "Block B" the author started postgraduate (doctoral) studies in 1976. It happened at Department of Strength of Materials of the same Technical University of Sofia, in the research line of fatigue life. The director of the doctoral studies was the department head Professor Petar Levchev Ganev.

After the literature review done it became apparent that nobody had searched for fatigue life by means of an integral of fatigue damage differentials. At the root of fatigue life knowledge the empirical relation lies which is known as *S-N* line or Wöhler line (August Wöhler, 1800 – 1882). All the following studies were mostly on it. The line represents an exponential relation between σ_a and N where σ_a is the amplitude of a sinusoidal cyclic stress $\sigma(t) \equiv \sigma_x(t) = \sigma_a \sin \omega t$ and N is number of cycles to fatigue rupture.

Yet, from the view point of the calculus, the *S-N* line i.e. the empirical relation $N = N(\sigma_a)$ can be treated as an integral result from summing some fatigue differentials per stress differentials $d\sigma$ during time differentials dt. The integration condition of this result is a simple specific case of cyclic σ variation. *However, no one had looked at the S-N line namely from this point of view*. Therefore, no question had been brought up for a differential $d\sigma$ entailing some fatigue differential so that the Wöhler relation $N = N(\sigma_a)$ would result from integration of such differentials under the condition $\sigma(t) = \sigma_a \sin \omega t$. If this had been done, other integrations would immediately have been also done under any other non-cyclic (non-periodical, non-sinusoidal) stress-time functions $\sigma(t)$. Instead, a different thing happened: after the pressing question arose of how to predict life under an arbitrary, non-cyclic, deterministic or random stress-time function $\sigma(t)$, *a process started for adapting the integral result from the case* $\sigma(t) = \sigma_a \sin \omega t$.

In other words, all the researchers directed themselves to looking for cycles with different amplitudes $\sigma_{a,i}$ *in a non-cyclic stress-time function* $\sigma(t)$. The latter was considered as a loading with a variable amplitude which accepted the different values $\sigma_{a,i}$, i.e. $\sigma(t)$ was replaced by a series of cycles with $\sigma_{a,i}$ amplitudes. The next adaptation to the *S-N* line was developed as follows. Researchers assumed that a so-called (relative) fatigue damage $1/N(\sigma_{a,i})$ occurs per one cycle with $\sigma_{a,i}$. The $N(\sigma_{a,i})$ life, shorter denoted as N_i, is taken from the *S-N* line. If the same cycle repeats itself to failure, the latter would occur in N_i cycles and the cumulative (relative) damage $D_\Sigma = \Sigma(1/N_i)$ would reach its full value 1 (i.e. 100 %). Thus, $n_i < N_i$ repetitions of $\sigma_{a,i}$ would make damage $n_i.(1/N_i) = n_i/N_i$. After summing such n_i/N_i damages from the different (grouped) $\sigma_{a,i}$ amplitudes, then the life is determinable from the equation $\Sigma(n_i/N_i) = 1$.

Such an approach (Subchapter 1.3) dates back to the 20s of XX century after a similar idea of Palmgren [138]. It was developed by Miner [129] in the 40s. That is why determination of the life so that $\Sigma(n_i/N_i) = 1$ is called the rule of Miner (or Palmgren-Miner).

Following the vein of the above treatise, the Miner rule is a way of adapting the integral result (the *S-N* line) under the particular integration condition $\sigma(t) = \sigma_a \sin\omega t$ to an arbitrary condition $\sigma(t)$. And, under such an interpretation, what happened later becomes already recognizable: a lot of methods were proposed for distinguishing cycles in a non-cyclic stress-time function and counting them. Such processing is also called schematization or decomposition of the non-cyclic stress-time function. In the last decades, the Rain-Flow Method of the CCA (the cycle counting approach) is the most popular [76] [84] (and many other publications). It was standardized in many countries, as well as in Bulgaria [1].

Except for the rain-flow method, in Europe there is also Eurocode 3:1993 Reservoirs Standard (according to a note of Prof. Lazov during discussions on this thesis on 21.06.2011).

Before the time of the analogue-to-digital converting and computers, special electronic counting apparatuses were created. Hundreds of papers were written on cycle counting problems.

In contrast to that all, following the above-mentioned author's setting for differentials and integrals, the sacred calculus equation $\mathrm{d}F(x) = f(x)\mathrm{d}x$ was addressed. Analogously and merely, the equation $\mathrm{d}D(\sigma) = R(\sigma)\mathrm{d}\sigma$ was built. Here, $\mathrm{d}\sigma$ is a differential of change of σ stress per time differential $\mathrm{d}t$; $D(\sigma)$ is (relative) fatigue damage which changes by differential $\mathrm{d}D(\sigma)$ per $\mathrm{d}\sigma$; $R(\sigma)$ is derivative of $D(\sigma)$ and as such is intensity of fatigue damage while σ undergoes $\mathrm{d}\sigma$. If integrating $\mathrm{d}D(\sigma) = R(\sigma)\mathrm{d}\sigma$ under the condition $\sigma(t) = \sigma_a \sin\omega t$ for one cycle i.e. for a time-period T, then a relative fatigue damage $D_{\Sigma,T}$ per one cycle will be obtained. And this $D_{\Sigma,T}$ is $1/N(\sigma_a)$ i.e. it relates to a point of the *S-N* line. Vice versa, if differentiating suitably this specific integral result $1/N(\sigma_a)$ as a primitive, then the derivative $R(\sigma)$ can be determined: like obtaining $f(x)$ in Eq. 0-2 by differentiating $F(x \equiv x_{max})$.

Hence, the determination of the damage intensity $R(\sigma)$ (*and thereafter operation on differential level of damage*) *can be done by relevant differentiation of a specific* $1/N(\sigma_a)$ *primitive represented by the S-N line. After that,* $\mathrm{d}D(\sigma) = R(\sigma)\mathrm{d}\sigma$ *can be integrated under any arbitrary stress-time function* $\sigma(t)$. *Thus, there is no need of preliminary distinguishing and counting of cycles i.e. no need of CCA that has been done for several decades. This is a radically different idea of using an S-N line from cyclic loadings for fatigue life evaluation under arbitrary non-cyclic loading.*

The σ stress above is meant to represent a uniaxial state of stress i.e. one-component loading. The development of the engineering in 20th century posed the question of the fatigue life under multiaxial stress at surface points where, besides $\sigma(t) \equiv \sigma_x(t)$, also $\sigma_y(t)$ and $\tau_{xy}(t)$ act. *In case the three $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$ oscillograms vary proportionally, the stressing is tantamount to one-component loading.* Indeed, it is again represented by a single variable. If the latter is denoted as s, then again it comes to a single $s = s(t)$ oscillogram. Again one S-N line can be used. The next problem is that, in the general case, the multiaxial (multi-component) loading is non-proportional: with three totally different $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$ oscillograms.

How did researchers start evaluating the fatigue life under multiaxial non-proportional loading? Again in the vein of the above treatise, what happened is expectable (Subchapter 1.4): without any loading and damage differentials defined in this general case, and therefore without any integration under arbitrary three-component conditions $\sigma_x = \sigma_x(t)$, $\sigma_y = \sigma_y(t)$ and $\tau_{xy} = \tau_{xy}(t)$, researchers started creating numerous methods. And, if under uniaxial loading many tens of criteria were proposed, then under two-component and three-component loading their number increased to the second and third power. It became very complicated while doing trials to generalize some results from one-component loading to two- or three-component loading. Such trials required certain concepts (theories) for fatigue equivalence between multiaxial non-proportional loading and one-component loading, respectively for reducing the loading multiaxiality. Many concepts (some of them are reviewed in Subchapter 1.3) and many corresponding conceptual problems appeared.

Whereas, *if integrating fatigue damage from a differential level of multiaxial stressing, directly under arbitrary $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$ oscillograms, then any necessity of reducing the loading multiaxiality drops out, neither is there any need of searching for cycles and counting them*. The mentioned conceptual problems do not appear.

But how to define a loading (stressing) differential (labeled again with $\mathrm{d}s$) under three components $\sigma_x(t)$, $\sigma_y(t)$ and $\tau_{xy}(t)$? Under one only component $\sigma(t) \equiv \sigma_x(t)$, the differential $\mathrm{d}s$ is quite simple according to above: it is $\mathrm{d}\sigma$. However, under three components, the definition of $\mathrm{d}s$ is not that simple at all: the three stresses are tensor-like dependent on (variant of) the choice of the x and y axes.

Hence *it is first to solve the problem of how to compose a three-component loading differential* d*s which is independent of x and y. The next problem*: *provided that* d*s is defined, how to compose the damage differential* d*D*?

These problems were a true challenge to the author's setting from the 'Block B' (and after some accumulated professional experience in algorithms and computer programming in a computer-processing center). A term of *trajectory* was introduced as a path described in the σ_x-σ_y-τ_{xy} coordinate system. It is obvious that while the describing running (current) point of the trajectory is going away from the coordinate origin, the damage intensity is rising. Then it stands to reason to associate the cumulative damage with accumulation of a (curvilinear) integral along the trajectory: the latter is composed by infinitesimal segments that are, in fact, stressing differentials d*s*. Per every d*s*, a damage differential d*D* is added with a damage intensity which steeply rises while σ_x, σ_y and τ_{xy} rise. However, this σ_x-σ_y-τ_{xy} trajectory is, as already understood, tensor-like *variant*, i.e. it will radically change if different *x* and *y* axes are chosen. Therefore another, *invariant* trajectory must be introduced. This is the trajectory of the principal stresses σ' and σ'' i.e. the trajectory described in the σ'-σ'' coordinate plane. However, the rotation of the principal axes ' and " would be omitted in this way. How should it be accounted?

After all, to the attention of the director Prof. Ganev and of the department, a three-component differential d*s* in the coordinate system ε'-ε''-dγ was submitted; ε' and ε'' are the principal strains, and dγ is a shear-strain differential. Addressing strains instead of stresses was influenced by Prof. Ganev who searched for application of an ε_x-ε_y hysteresis loop discovered by him [7]. The idea of d*s* in ε'-ε''-dγ coordinates remains the same in σ'-σ''-dτ coordinates as presented in this thesis (Subchapter 2.1).

The director and the department took intense interest in the differential d*s* proposed. The multiaxial (three-component) loading could already be represented as a multitude of d*s* differentials. They are technically formed as finite differences that are short enough and of a sufficiently great number. Thus IDD was formed as a numerical method enabled only by means of a computer. By the way, for the necessity of a computer, the IDD method would not have been proposed for practical application earlier than e.g. 1970. In fact, the method hit upon the beginning of the mass computerization and the entailed possibility of numerical differentiation and integration in a large volume. On this basis, also in other scientific fields, methods were developed that had been unthinkable before. And, finally, contemporary kinds of software appeared for finite elements (FE) modeling.

By the way again, particularly under uniaxial arbitrary non-cyclic loading, there had been no technical obstacles to IDD: it could have been developed and applied (instead of CCA) much earlier, e.g. in the beginning of 20th century. Under such loading, the integration can be done without any computer (it is shown how in Subchapter 2.3).

The idea of $\mathrm{d}s$ and the first computer programming already done were published [14] 1978. This is the registered beginning of the IDD method (as discussed below, the name 'IDD' was accepted 2009 and it substituted the previous name 'Integral Method'). After $\mathrm{d}s$, the damage differential $\mathrm{d}D$ was also postulated, first in the simplest way from above: $\mathrm{d}D(s) = R(s)\mathrm{d}s$. This was a hypothesis that the same damage intensity $R(s)$ [13] (1979) could be used under different integration conditions. This hypothesis was confirmed in the ε'-ε''-$\mathrm{d}\gamma$ coordinate system by using other authors' experimental data and later personal data [34] under non-proportional bending and torsion. A lot of efforts went on programming for the then computers, by means of which the method could work, and on making a testing machine to carry out personal experimental data. The dissertation [41] was successfully defended 1980.

The director envisaged a great future for IDD in combination with input S-N lines obtained in an accelerated manner based on his ε_x-ε_y hysteresis loop [7]. He procured a job position for the author to the department's scientific stuff. Dissemination of the Integral Method started within Bulgaria [10] [38] and the former Soviet Union [11]. Prof. Ganev was an influential scientific authority in Bulgaria. Unfortunately, he fell ill 1984 and passed away 1985. The new department head, Prof. Stoyan Nedelchev, did such a communist policy that the final result for the author was leaving the Technical University. The author's career continued at University of Forestry, Faculty of Forest Industry, in different scientific research lines. The development of the Integral Method broke until the author became an associate professor 1991.

At that time Bulgaria opened itself to the world and the author returned to the method in order to popularize it internationally. It seemed that the very idea to sum fatigue damage differentials directly under any loading, without any cycle counting or reducing stress multiaxiality, would find the same respect and support as in the Department of Strength of Materials 1978 - 1984. Introducing the damage differentials and an integral of them seemed to have the same revolutionary importance for the fatigue life research like the importance of introducing differentials and integrals into the mathematics and related exact sciences. It seemed that the Integral Method would immediately be taken up from the world fatigue life research authorities and institutions.

In 1993 there was a two-month author's study visit to the one of European centers of fatigue research: the Prof. Zenner's IMAB-institute in Clausthal, Germany. Prof. Zenner, as one of the world fatigue research authorities, and his collaborators showed some interest to the Integral Method. But they only wished success in its development: they had their own scientific program and financing, and each colleague had his own task.

Meanwhile, seeing the wide spreading of personal computers, the author was busy in strenuous re-programming the method, including also graph mode. The algorithm (Chapter 4) is not easy. Its approbations, finding out the inevitable programming mistakes and their cleaning out, took much effort (from a single person) of many years.

The author was successful to have four papers [169] [170] [171] [176] published in *Int. J. Fatigue* 1993 – 1997. Each publication took one-year effort. The referees showed reserves about the unknown author from Bulgaria who tried to propose something nontraditional. The fourth paper [169] was dedicated to the interesting characteristic loading case in which the principal axes rotate but the principal stresses remain constant (Sections 2.6.3 and 2.6.4). This case was revealed thanks to IDD and the paper emphasized that it remained undeservedly unnoticed; that it is very important because it is a “maximized” case. A lot of fatigue life criteria should be approbated under such loading to check their validity. Then, many of them would fail.

After this fourth paper, *Int. J. Fatigue* did not admit to publication a fifth paper submitted. In it, the point was openly set that the whole world fatigue life evaluation experience should be redirected to relations on differential level from where integration should be done as a uniform method under any sorts of loading. Another paper, ‘Fatigue Life Prediction without Cycle Counting (Using an Integral)’, was also denied. A situation became apparent that the Integral Method is not accepted by the authors and supporters of the many existing CCA methods established also by government standards.

But later, already on a regional level in Bulgaria, the paper ‘Fatigue Life Prediction without Cycle Counting (by Means of the Integral Method)’ was accepted and published in the *J. Theoretical and Applied Mechanics* (of the Bulgarian Academy of Sciences) [165].

In the meantime, Prof. Ewald Macha from the Technical University of Opole, Poland, took an interest in the Integral Method. This stimulated building a team with the author leading and with participation of Prof. Macha and his Polish collaborators for doing "Development of the Integral Method for Fatigue Life Prediction under Multiaxial Non-proportional Arbitrary or Random Loading". Under this title, the Bulgarian Fund of Scientific Investigations granted some financing ('TH-545/95' contract with the University of Forestry). However, for the post-communist crisis and inflation in Bulgaria at that time, the resources quickly exhausted. Besides, a negative situation occurred in the relations with the Polish colleagues: they presented oscillograms in which the IDD graph mode discovered parasite impulses. This concerned a large volume of experimental data. The author created additional software which cleaned out ('shaved') the impulses [45] [46]. However, the Polish colleagues preferred to withdraw from the collaboration (besides, IDD provoked the question of how correct the concept of their method is).

Another stimulus appeared relating to Prof. De Mare and his collaborators from Sweden. It turned out [103] [104] [182] that they, independently and in a later time, enabled fatigue life prediction under one single oscillogram by using its instantaneous ordinates instead of amplitudes, what is the same with the Integral Method. Respectively, they also defined damage intensity although they did not call it so: it is their function $g(s)$ which equals the IDD $R(s)$ intensity in the particular case of zero static level of the oscillogram. They had not initiated the idea of loading and damage differentials and correspondingly they did not talk about an integral of such differentials.

Prof. De Mare was contacted for cooperation. An invitation followed and financing on Swedish part was provided to the author for a study visit to Sweden and participation in the Workshop "Statistical Methods in Fatigue of Materials" 1998. A talk was given [164] in which the Integral Method was briefly represented and its point of intersection with the Swedish authors' model was shown. A call was extended for joint effort for a new approach to fatigue life prediction starting from differential level. The response was reserved. After all, the Swedish colleagues did not show any intention to generalize their method to something more than its original direction.

It became more and more apparent that whether the Integral Method would really be the right new approach or not is on the one hand only. On the other hand the circumstance was that a global acknowledgment of the Integral Method would require from the other authors to reevaluate, readjust or even deny their own concepts. However, they made their careers and obtained finances thanks to their concepts.

Hence, the reserved attitude or even a preliminary negative aptitude to IDD seems to be logical. A new global approach would be welcomed if it was advanced by a world-famous luminary in the research field and supported by a powerful financial institution. The author nearly gave up next trials.

In 1998, a second two-month study visit to the Prof. Zenner's institute, Germany, was enabled again. A touch with the University of Braunschweig was enabled, as well, and a talk was also given there. Any persistent IDD followers were not found.

Within the period 1999 – 2003, the author was to the US for two years and a half. In 2002, he was a visiting professor at the Illinois Institute of Technology, Department of Mechanical, Materials and Aerospace Engineering (MMAE). Some trials were done to engage MMAE and other American colleagues with the Integral Method. But they kindly denied for being busy in their own tasks and projects.

Thus, the work on the Integral Method broke again. Subsequently, the method proved to be already well-known and discussed in the world: [62] [63] [99] [102] [107] [144] [145] etc. There was the acknowledgement [63] that the IDD concept is far beyond the scope of the previous studies. There were remarks, as well. They additionally showed that the work must continue, and cooperation and followers should be found. Besides, an IDD public defense procedure should be evoked as an additional way to engage the attention and valuation of more people.

A next stimulus appeared again. It came on the part of a doctorand (postgraduate student), assistant professor Boyan Stoychev, from the Department of Engineering Mechanics at the Technical University of Gabrovo, Bulgaria. Cooperation started for building a new testing machine for rotating bending combined with constant torsion [179] designed on the basis of an author's scheme [39] [40] (Section 1.4.6). The experimental data obtained by this machine served for a successful IDD verification (in Chapter 5).

A Bulgarian IDD site, http://MetodNaIntegrala.hit.bg, was created in 2006. It is continuously maintained and expanded up to present. The method is popularized there in Bulgarian language. This thesis is also exposed there together with computer programs and files. As well, a Volume II is exposed containing expansions, supplements, details, etc. References to the IDD site are done for everything which belongs to this thesis or is its continuation but cannot be included here due to the limited volume.

In English, the same site is http://www.freewebs.com/fatigue-life-integral. It became main means for popularizing IDD abroad and for establishing contacts with many colleagues throughout the world.

As a result, collaboration with Dr. Jan Papuga and his colleagues from Czech Republic was established. Dr. Papuga is a young scientist who has a present and a future of a world authority on fatigue of materials, mainly for a site he had created: http://www.pragtic.com/. There, an ambitious so-called PragTic Project is exposed. It contains a large fatigue strength data bank, a lot of methods and software, communication in a PragTic society, PragTic forum, organization of regular annual PragTic conferences, and so on. On his site, Dr. Papuga proclaimed the integration of damage differentials without forming any cycles to be a revolutionary idea (http://www.pragtic.com/docu/PragTicA_Intro.pdf, p. 9).

From the collaboration with Dr. Papuga, the paper [174] resulted. It was reported on an international conference in Darmstadt, Germany. In this paper, the IDD abbreviation was suggested and accepted as a better name. The reported IDD lives under one-component random loadings computed without using the rain-flow procedure (which otherwise was expected on the conference) proved to be the most accurate (Subchapter 3.3).

In the summer 2009, a third two-month study visit to Germany was done, this time to the Fraunhofer Institute for Structural Durability and System Reliability LBF (Fraunhofer-Institut für Betriebsfestigkeit und Systemzuverlässigkeit LBF), at the invitation of Prof. C. M. Sonsino. He is one of the world-famous scientists in fatigue of materials. Thanks to him and to his interest in IDD, this visit was very fruitful. The study [168] (70 pages), with the participation of Prof. Sonsino and the LBF director, Prof. H. Hanselka, was written. This is the LBF 2009 annual report which took the character of a monograph on IDD. As well, the papers [167] and [175] devoted to IDD verifications were written with participation of other more LBF colleagues.

The activity through the IDD site and the interest in IDD led to including the author in the Scientific Committee of the ICMFF9 (the Ninth International Conference on Multiaxial Fatigue & Fracture), Parma, Italy, June 7 – 9, 2010 (http://www.icmff9.unipr.it/). This conference is held once three years. The most known researchers in the subject come together to this conference and it is the most relevant forum for IDD. The author actively participated in the conference and reported an invited paper [166]. Discussions on IDD and engaging the ICMFF9 audience attention to IDD were evoked. Other more IDD papers [167] [175] [178] were included in the ICMFF9 proceedings.

In the person of the ICMFF9 co-chairmen, Prof. Andrea Carpinteri from the University of Parma and Prof. Sonsino, the author found the long-expected acknowledgement and support for the importance of IDD. On their initiative, an invitation came for IDD publication in a special issue of *J. Fatigue* 2011 devoted to ICMFF9. All this turned into a good reason for finalizing and presenting this thesis.

To the attention of researchers studying the method proposed

It is expected that researchers studying and evaluating this thesis would take up attitude toward it in the next succession (to acknowledge or not): 1 – the new research line, i.e. the IDD concept appeared in Bulgaria; 2 – the loading differential $\mathrm{d}s$ proposed for the general case of loading (stressing); the $\mathrm{d}D$ integrand proposed for the general case together with an empirical data bank of IDD parameters participating in the integrand.

On the point 1, a firm conviction is expressed here that the IDD concept is definitely necessary (regardless of whether it is implemented by the method proposed or by a possible future better method). Moreover, the IDD concept absence before was some mathematical gap of the fatigue life researchers. IDD carries the basic idea of the calculus to integrate from differential level under arbitrary integration conditions which, in the case considered, are arbitrary variations of the fatigue loading components. In this way, conceptually and universally the most important fatigue life assessment problem is solved: the influence of the different kinds of loading. Newton and Leibniz would congratulate this approach.

On the point 2, i.e. on the issue of $\mathrm{d}s$ definition, it is to claim that a certain achievement has been realized, and that the absence of such $\mathrm{d}s$ definition before was possibly the reason for the absence on the point 1. The logic of the $\mathrm{d}s$ definition proposed seems to be solely possible. Any other researcher can try to formulate a different loading differential. If failing, then the $\mathrm{d}s$ differential proposed here will be established as the only possible one.

On the point 3: the $\mathrm{d}D$ integrand proposed (in Subchapter 2.7), i.e. the author's IDD method, works successfully. This is confirmed by the verifications done in Chapter 5. In the IDD method and software proposed, empirical IDD parameters denoted as f_c, f_τ, N_c and N_τ are used; f_c and f_τ are called factors of (sensitivity of the material to) loading non-proportionality; the N_c and N_τ numbers serve for forming areas of no damage in the σ'-σ'' plane valid under non-proportional loading. The method works with an accuracy which corresponds to the present stage of managing the parameters f_c, f_τ, N_c and N_τ. This accuracy is quite satisfactory against the background of the too many and contradictory existing methods.

More on the preview of the method can be seen on the IDD site (on the Home page, 'More on the method', or on http://www.freewebs.com/fatigue-life-integral/More.pdf).